PART C —
$$(3 \times 10 = 30 \text{ marks})$$

Answer any THREE questions.

- 16. (a) From the set of vectors (1,0,1), (0,0,1) and (1,1,0) construct a set of orthonormal vectors.
 - (b) State and prove the expansion theorem in linear vector space.
- 17. State and prove the Cauchy Residue theorem and then find the residues of $f(z) = \frac{ze^{i\theta}}{z^4 + a^4}$ at its poles.
- 18. Find the eigen-values and the normalized eigen vectors of the following matrices

(a)
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$
 (b)
$$\begin{bmatrix} 5 & 0 & \sqrt{3} \\ 0 & 3 & 0 \\ \sqrt{3} & 0 & 3 \end{bmatrix}$$

- 19. Find the Fourier transform of the Gaussian distribution function $f(x) = Ne^{-\alpha x^2}$ Where N and a are constants.
- 20. Prove that

(a)
$$e^{2zx-z^2 = \sum_{n=0}^{n=\infty} \frac{H_n(x)}{n!} z^n}$$

(b)
$$2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$$
.

23PPH11 — MATHEMATICAL PHYSICS

Time: Three hours

Maximum: 75 marks

PART A —
$$(10 \times 2 = 20 \text{ marks})$$

Answer ALL questions.

- 1. Define Subspaces of vector spaces.
- 2. Show that the vectors (1,2,-3), (1,3,-2) and (2,-1,5) are linearly independent.

What is singular point of analytic function?

Find the poles for the function $f(z) = \frac{z}{\cos z}$.

- Find the inverse of the matrix for $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}.$
- 6. Show that the eigen values of a Hermitian matrix are real.
- 7. State linearity property of Laplace transform.

- 8. Expand the function $f(x) = \sin x$ as a cosine series. in the interval $(0, \pi)$.
- 9. If $P_n(x)$ is a Legendre polynomial, then value of $\int_{-1}^{+1} [p_n(x)]^2 dx$ is.
- 10. Define Generating function.

PART B —
$$(5 \times 5 = 25 \text{ marks})$$

Answer ALL questions.

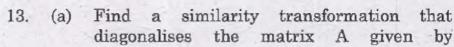
11. (a) Describe the Schmidt's orthogonalization process in some n-dimensional vector space.

Or

- (b) Obtain an orthogonal basis for the subspace of \mathbb{R}^4 spanned by $X_1 = (1,0,1,0)$, $X_2 = (1,1,1,1)$, $X_3 = (-1,2,0,1)$.
- 12. (a) Show that the real and imaginary parts of the function log z satisfy the Cauchy-Reimann equations, when z is not zero.

Or

(b) Expand the function as a Taylor's series $f(z) = \frac{1}{z+1}$ about z = 1,



$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

Or

If
$$A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$
, Show that

 $A^{-1} = A^{T}, A^{T}$ being a transpose of matrix A.

14. (a) Prove the similarity theorem or change of scale property from $g(\omega)$ is the Fourier transform of f(t), the Fourier transform of f(at) is $\frac{1}{a}g\left(\frac{\omega}{a}\right)$.

Or

- (b) Find the inverse Laplace transform of In $\left(\frac{s^2+w^2}{s^2}\right)$.
- 15. (a) Derive the orthogonality relation $\int_{-1}^{+1} p_n(x) p_m(x) dx = 0 \text{ if } m \neq n.$

Or

(b) Find the derivatives of Dirac delta function at the origin x = 0.

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